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ASYMPTOTIC OF A SLIGHTLY VISCOUS FLUID FLOW UNDER THE EFFECT OF TANGENTIAL STRESSES ON A FREE BOUNDARY

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UDC 532.526

Formal asymptotic expansions of the solution of a plane nonlinear stationary problem with a free boundary are constructed for high Reynolds numbers under the assumption that the surface tangential stresses are given and have a finite value. The boundary layer equations near the free boundary are nonlinear, while the principal terms of the asymptotic outside the boundary layer satisfy the Euler ideal-fluid equations. It is shown that the action of the tangential stresses results in the appearance of an additional term equivalent to the surface tension forces in the dynamic boundary condition on the free boundary of a "limit" inviscid flow.

1. A plane nonlinear stationary problem on the motion of an incompressible fluid is considered for the Navier–Stokes equations with vanishing viscosity ($\nu \to 0$) in a domain D bounded by the free surface Γ subjected to tangential and normal stresses given on Γ:

$$(\mathbf{v},\ \nabla)\mathbf{v} = -\rho^{-1}\nabla p + \nu\Delta\mathbf{v} + \mathbf{g},\ \operatorname{div}\mathbf{v} = 0; \quad (1.1)$$

$$p - 2\rho\nu\partial v_n/\partial n = p_* + \varkappa\sigma, \quad \rho\nu\mathbf{n}(\boldsymbol{\tau}\cdot\nabla)\,\mathbf{v} = T, \quad (x,\ z) \in \Gamma; \quad (1.2)$$

$$\mathbf{v}\cdot\mathbf{n} = 0,\ (x,\ z) \in \Gamma. \quad (1.3)$$

Here $\mathbf{v} = (v_x,\ v_z)$, $\mathbf{g} = -g\mathbf{e}_z$, $\mathbf{e}_z = (0,\ 1)$ is the direction of the z axis, ρ is the fluid density, g is the acceleration of gravity, σ = const > 0 is the coefficient of surface tension, κ is the curvature of the free boundary Γ (κ > 0 if Γ is convex outside the fluid); **n** and **τ** are unit vectors of the external normal and the tangent to Γ; p_* is the given pressure on Γ; and T is the tangential stress on Γ [T = O(1) as $\nu \to 0$]. It is assumed that the domain D is not bounded and the behavior of the velocity field at infinity is given.

Rostov-on-Don. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 101-107, September-October, 1987. Original article submitted June 9, 1986.

0021-8944/87/2805-0737$12.50

For vanishing viscosity ($\nu \to 0$) near the free boundary a boundary layer of infinitely large vorticity of the order of $O(1/\nu)$ is formed. In the external domain (outside the boundary layer) the flow is described approximately by the Euler equations. In the case of no tangential stresses ($T = 0$) on the free boundary asymptotic expansions of the solution of the problem are constructed in [1], where it is shown that the boundary layer equations are linear and solved in quadratures. For $T = O(1)$ near Γ a boundary layer can occur that satisfies the nonlinear equations. Equations of the Marangoni boundary layer that occurs near the free boundary of a nonuniformly heated fluid because of the thermocapillary effect are formulated in [2] and studied in [3-5].

The asymptotic is studied below for a low-viscosity fluid flow that satisfies the system (1.1)-(1.3). The problem (1.1)-(1.3) is reduced to dimensionless form and a small parameter $\varepsilon = \nu\rho^{1/2}L_*^{-1}T_*^{-1/2}$ is introduced (L_*, $T_*^{1/2}\rho^{-1/2}$, T_* are the length, velocity, and tangential velocity scales). The asymptotic expansions of the solution of the problem as $\varepsilon \to 0$ are constructed in the form

$$\mathbf{v} \sim \frac{1}{\varepsilon^{1/3}}\mathbf{h}_0 + \sum_{k=0}^{N} \varepsilon^{k/3}(\mathbf{v}_k + \mathbf{h}_{k+1}), \tag{1.4}$$

$$p \sim -z + \sum_{k=0}^{N} \varepsilon^{k/3}(p_k + q_k), \quad \zeta \sim \sum_{k=0}^{N} \varepsilon^{k/3}\zeta_k,$$

where $z = \zeta(x)$ is the equation of the free boundary. Let D_Γ denote the boundary layer domain. Then $\mathbf{h}_k$, q_k are functions of the type of the boundary layer problem in D_Γ. The vector-functions $\mathbf{v}_k$ and the functions p_k determine the asymptotic solution of the problem everywhere outside D_Γ. Let us note that the order of the principal term in the expansion of the velocity vector in (1.4) and the order of the boundary layer thickness are found from the condition that the orders of the viscous and inertial terms in the Navier–Stokes system (1.1) as well as in the boundary conditions (1.2) for the tangential stress $\partial\mathbf{v}/\partial n = O(1)\varepsilon$ as $\varepsilon \to 0$ are equal. The boundary layer thickness is of the order of $\varepsilon^{2/3}$ in this case.

Let us formulate the problem for the principal terms $\mathbf{h}_0$ and q_0 of the asymptotic (1.4) that govern the flow in the boundary layer domain. Near Γ we introduce the local orthogonal coordinates (r, φ) by means of the formulas

$$x = X(\varphi) - rn_x, \; z = Z(\varphi) - rn_z.$$

Here r is the distance between the point (x, z) and the contour Γ; n_x, n_z are components of the unit vector normal to Γ drawn within the domain D; and $x = X(\varphi)$, $z = Z(\varphi)$ are the parametric equations of the contour Γ. For sufficiently small r segments of the normal of length r are not intersected.

The boundary value problem for $\mathbf{h}_0$ is derived by applying the second iteration process [6] to the system (1.1)-(1.3). Let $h_{\varphi k}$, h_{rk}, and $v_{\varphi k}$, v_{rk} be components of the vectors $\mathbf{h}_k$ and $\mathbf{v}_k$, respectively, in the coordinates (r, φ). We substitute the expansions (1.4) into (1.1), (1.3) and into the dynamical condition (1.2) for the tangential stress on Γ. Let us expand $\mathbf{v}_k$ and p_k in a Taylor series in powers of r and let us set $r = \varepsilon^{2/3}s$. We introduce the notation $H_{r2} = h_{r2} + v_{r1}|_{r=0}$. Equating the coefficients of ε^{-1}; $\varepsilon^{-2/3}$ to zero we find $h_{r0} = h_{r1} = 0$, while we deduce the boundary value problem for $h_{\varphi 0}$, H_{r2}

$$\frac{1}{\delta}h_{\varphi 0}\frac{\partial h_{\varphi 0}}{\partial \varphi} + H_{r2}\frac{\partial h_{\varphi 0}}{\partial s} = \frac{\partial^2 h_{\varphi 0}}{\partial s^2}, \quad \frac{\partial h_{\varphi 0}}{\partial \varphi} + \delta\frac{\partial H_{r2}}{\partial s} = 0, \tag{1.5}$$

$$\left.\frac{\partial h_{\varphi 0}}{\partial s}\right|_{s=0} = -T(\varphi), \quad H_{r2}|_{s=0} = 0, \quad h_{\varphi 0}|_{s=\infty} = 0,$$

where δ is the Lamé coefficient of the curve $\Gamma(\delta^2 = (\partial X/\partial\varphi)^2 + (\partial Z/\partial\varphi)^2)$. The problem (1.5) on the segment $\varphi \in [0, l]$ is studied in [7] for a given initial profile $h_{\varphi 0} = f(s)$ ($\varphi = 0$), where the solvability conditions are found $T(\varphi) \in c^1[0, \ell]$, $f(s) \in c^{2+\alpha}[0, \infty]$, $\alpha > 0$, $T(0) = -f'(0)$, $f(s) \to 0$ ($s \to \infty$), $f(s) > 0$, $T(\varphi) \geq 0$.

The higher approximations of $\mathbf{h}_k$ ($k \geq 1$) are found by solving the linear boundary value problems

$$\frac{1}{\delta}h_{\varphi 0}\frac{\partial h_{\varphi k}}{\partial \varphi} + \frac{1}{\delta}\frac{\partial h_{\varphi 0}}{\partial \varphi}h_{\varphi k} + H_{r,k+2}\frac{\partial h_{\varphi 0}}{\partial s} + H_{r2}\frac{\partial h_{\varphi k}}{\partial s} = \frac{\partial^2 h_{\varphi k}}{\partial s^2} + F_k, \quad \frac{1}{\delta}\frac{\partial h_{\varphi k}}{\partial \varphi} + \frac{\partial H_{r,k+2}}{\partial s} = N_k,$$

$$\left.\frac{\partial h_{\varphi k}}{\partial s}\right|_{s=0}=M_k,\quad H_{r,k+2}|_{s=0}=G_k,\quad h_{\varphi k}|_{s=\infty}=0$$

($H_{r,k+2} = h_{r,k+2} + v_{r,k+1}$). The coefficients F_k, N_k, M_k, G_k are known and not written down because of their awkwardness, where $F_1 = N_1 = M_1 = G_1 = 0$ for $v_0 = 0$.

Now, applying the second iteration process to the Navier–Stokes system (1.1) projected on the normal to the free boundary, we derive an equation for the correction to the pressure q_0 in D_Γ, from which there follows

$$q_0=-\varkappa\int\limits_s^\infty h_{\varphi 0}^2 ds. \tag{1.6}$$

Let us find the value of q_0 on the free boundary. Integrating the boundary layer equation in (1.5) with respect to s on the semiaxis $(0, \infty)$, we apply integration by parts by taking account of the condition on Γ and by integrating the equation obtained with respect to φ we derive the relationship

$$\int\limits_0^\infty h_{\varphi 0}^2 ds=\int\limits_0^\infty f_0^2(s)\,ds+\int\limits_{\varphi_0}^{\varphi} T\delta d\varphi. \tag{1.7}$$

Here $f_0(s) = h_{\varphi 0}(s, \varphi_0)$ is the velocity profile in the boundary layer in the section $\varphi = \varphi_0$. Setting $s = 0$ in (1.6) and taking account of (1.7) we have

$$q_0=-\varkappa\left(\int\limits_{\varphi_0}^{\varphi} T\delta d\varphi+\int\limits_0^\infty f_0^2(s)\,ds\right)\quad (s=0). \tag{1.8}$$

If the velocity profile in the boundary layer is known for a certain φ_0, then the value of the boundary layer correction to the pressure on the free boundary is determined without solving the problem for the boundary layer (1.5).

Let us present the self-similar solution of the problem (1.5) for the case when the tangential surface stress is given by the power law $T = \tau_0\varphi^n$. The self-similar solutions of the more general equations of the stationary Marangoni boundary layer are constructed in [3]. Let φ be the arclength of the contour Γ, then $\delta = 1$. Let us introduce the stream function ψ by means of the formulas $h_{\varphi 0} = \partial\psi/\partial s$, $H_{r2} = -\partial\psi/\partial\varphi$ and let us use the notation $\eta = s\varphi^{(n-1)/3}$. Representing ψ in the form $\psi = \varphi^{(n+2)/3}g(\eta)$ we derive a boundary value problem for the function $g(\eta)$ from (1.5) (it is not necessary to give the initial profile since it is determined by the self-similarity condition)

$$g'''+\frac{n+2}{3}gg''-\frac{2n+1}{3}g'^2=0,\ g(0)=0,\ g''(0)=-\tau_0,\ g'(\infty)=0. \tag{1.9}$$

The relationship (1.7) results in the condition $\tau_0(n + 1) > 0$. The function $g(\eta)$ is constructed numerically for different n. Thus for $n = 0$ and $\tau_0 = 1$ the function $g(\eta)$ grows monotonically from zero to $g(\infty) = 1.481$ on the semiaxis $(0, \infty)$. For $n = 1$ an exact solution $g=\sqrt[3]{\tau_0}\left[1-\exp\left(-\sqrt[3]{\tau_0}\eta\right)\right]$ exists, where τ_0 is positive. For $n = -2$ the equation (1.9) has a power-law solution $g=\dfrac{6\sqrt[3]{-\tau_0}}{\eta\sqrt[3]{-\tau_0}+\sqrt[3]{12}}-\dfrac{6\sqrt[3]{-\tau_0}}{\sqrt[3]{12}}$. Here τ_0 is negative.

We now present the boundary value problems for v_k, p_k that govern the flow outside the boundary layer domain. The equations for v_k, p_k are obtained by applying the first iteration process [6] to the Navier–Stokes system. Let us transfer all the terms of the system (1.1) to the left side, which we will denote by P(V), where $V = (v_x, v_x, p)$. Furthermore, we require satisfaction of the relationships

$$P(\mathbf{V}_N)=O(\varepsilon^{(N+1)/3}),\ \mathbf{V}_N=\left(\sum_{k=0}^N\varepsilon^{k/3}\mathbf{v}_k,\quad\sum_{k=0}^N\varepsilon^{k/3}p_k\right). \tag{1.10}$$

Equating the sum of the coefficients of $\varepsilon^{k/3}$ successively to zero in (1.10), we obtain a system of equations to determine v_k, p_k

$$\sum_{i+j=k} (\mathbf{v}_i, \nabla)\mathbf{v}_j = -\nabla p_k + \Delta \mathbf{v}_{k-3}, \qquad (1.11)$$
$$\operatorname{div} \mathbf{v}_k = 0 \ (k = 0, 1, \ldots, N;\ v_{-1} = v_{-2} = v_{-3} = 0).$$

We derive the boundary conditions for the systems (1.11) by applying the first and second iteration processes simultaneously to (1.2) and (1.3). We let Γ_0 denote the free boundary of an inviscid flow $\mathbf{v}_0$, p_0. Near Γ_0 we introduce the local orthogonal coordinates (r_1, φ_1) (r_1 is the distance to Γ_0). We represent the curvature of the curve Γ in the form $\kappa = \kappa_0 + \varepsilon^{1/3}\kappa_1 + \ldots$ (κ_0 is the curvature of the contour Γ_0). We substitute the expansion (1.4) into (1.3) and in the dynamic condition for the normal stress in (1.2) and we go over to the coordinates (r_1, φ_1). Equating coefficients of $\varepsilon^{k/3}$ to zero, we deduce boundary conditions for the systems (1.11), which we write in the dimensionless form

$$\mathbf{v}_k \cdot \mathbf{n}_0 = -H_{r,k+1} + E_k,\ p_k + q_k + \zeta_k \frac{\partial p_0}{\partial r_1} - \frac{\varkappa_k}{\mathrm{Bo}} = R\ ,\ (x, z) \in \Gamma_0. \qquad (1.12)$$

where $\mathbf{n}_0$ is the unit vector of the external normal to Γ_0; $\mathrm{Bo} = \rho g L_*^2/\sigma$ is the Bond number, and $E_0 = R_0 = 0$. The coefficients E_k, R_k ($k \geq 1$) are not written down because of their awkwardness. In the coordinates r_1, φ_1 the coefficient $\zeta_0 = 0$, since $r_1 = 0$ is the equation of Γ_0.

The principal terms $\mathbf{v}_0$, p_0 of the asymptotic expansions (1.4) that govern the flow of an ideal fluid with free boundary Γ_0 are found from (1.11) and (1.12) for $k = 0$ with (1.8) taken into account, and they satisfy the boundary value problem

$$(\mathbf{v}_0, \nabla)\mathbf{v}_0 = -\nabla p_0 - \mathbf{e}_z,\ \operatorname{div} \mathbf{v}_0 = 0, \qquad (1.13)$$
$$\mathbf{v}_0 \cdot \mathbf{n}_0 = 0,\ p_0 = p_* + \frac{\varkappa_0}{\mathrm{Bo}} + \varkappa_0 \left(\int_{\varphi_0}^{\varphi} T\delta d\varphi + \int_0^{\infty} f_0^2(s)\, ds \right),\quad (x, z) \in \Gamma_0.$$

Therefore, the action of the tangential stresses on the free boundary of a low viscosity fluid result in the appearance of an additional term corresponding to the ideal fluid flow (1.13) in the dynamic boundary condition on the free boundary, and which can be interpreted as the action of capillary forces with a variable coefficient of surface tension.

2. We examine the case of no velocity field in an inviscid fluid. Let us determine the shape of the free surface of a low viscosity fluid when the ideal fluid is at rest ($\mathbf{v}_0 = 0$ and $p_* = 0$). Here $\mathbf{h}_1 = 0$. It follows from (1.13) that the free boundary Γ_0 written in dimensional variables will satisfy the equation

$$\varkappa_0 \left(\sigma + \gamma + \int_{\varphi_0}^{\varphi} T\delta d\varphi \right) = \rho g z + c, \quad c = \mathrm{const}, \qquad (2.1)$$
$$\gamma = \int_0^{\infty} f_0^2(s)\, ds = \mathrm{const},$$

which has a solution in quadratures in the absence of gravity forces ($g = 0$). Let φ be the arclength of the contour Γ_0; then $\delta = 1$. Let us write (2.1) in parametric form. We represent the equation of Γ_0 in the form $x = x(\varphi)$, $z = z(\varphi)$. We let $\beta(\varphi)$ denote the slope of an element of the line Γ_0, obtained as φ increases to the Ox axis. Then $x' = \cos\beta$, $z' = \sin\beta$. The equation of the boundary Γ_0 takes the form

$$\left(\sigma + \gamma + \int_{\varphi_0}^{\varphi} T d\varphi \right) x'' = \mp c z', \quad \left(\sigma + \gamma + \int_{\varphi_0}^{\varphi} T d\varphi \right) z'' = \pm c x'$$

where the upper or lower sign is selected in conformity with whether the fluid is located above or below the surface Γ_0 relative to the Oz axis. It is easy to show that $\beta(\varphi)$ satisfies the equation

$$\frac{d\beta}{d\varphi}=\frac{\pm c}{\sigma+\gamma+\int\limits_{\varphi_0}^{\varphi} T d\varphi}.$$

Now having determined $\beta(\varphi)$ we find the equation of Γ_0

$$x=c_1+\int\limits_0^{\varphi}\cos\beta d\varphi,\quad z=c_2+\int\limits_0^{\varphi}\sin\beta d\varphi, \tag{2.2}$$

$$\beta=c_3\pm c\int\limits_0^{\varphi}\frac{d\varphi}{\sigma+\gamma+\int\limits_{\varphi_0}^{\varphi} T d\varphi}.$$

Here c_1, c_2 determine the Cartesian coordinates of the reference point B of the arclength φ, while the constant c_3 is the slope of the tangent to Γ_0 at the point B. The constant c is found from additional conditions in each specific case.

Example 1. Let the fluid fill a semiinfinite strip $-\infty \le z \le \zeta(x)$, $0 \le x \le L$ bounded by the solid walls $x = 0$, $x = L$ and the free surface Γ. A constant tangential stress $T =$ const > 0 acts on Γ. We start to measure the parameter φ from the wall $x = 0$ where the positive direction of the coordinate φ is selected to agree with the direction of the tangential stress. The boundary layer equations (1.5) have the solution $h_{\varphi 0} = \varphi^{1/3}g'(\eta)$, where $\eta = s\varphi^{-1/3}(g(\eta))$ is determined numerically from the boundary value problem (1.9) for $n = 0$. In this case $\varphi_0 = 0$, $f_0(s) = 0$, and the constant is $\gamma = 0$. Equation (2.2) is reduced to the form

$$x=c_1+\frac{\sigma+T\varphi}{T^2+c^2}(T\cos\beta+c\sin\beta),\ z=c_2+\frac{\sigma+T\varphi}{T^2+c^2}(T\sin\beta-c\cos\beta), \tag{2.3}$$

$$\beta=c_3+\frac{c}{T}\ln(1+T\varphi/\sigma).$$

We express the constants c_1, c_2, c in terms of values of the angles formed by the boundary Γ_0 with the solid walls at the contact points as well as in terms of the Cartesian coordinates for the reference point of the variable φ. Let $x = 0$, $z = 0$, $\beta = \beta_0$ for $\varphi = 0$ and $\beta = \beta_1$ for $x = L$. We reduce the system of equations for c, c_1, c_2, c_3 to a nonlinear algebraic equation for the arclength φ_1 of the contour Γ_0

$$\left[1+\frac{\beta_1-\beta_0}{\ln^2\left(1+\frac{T\varphi_1}{\sigma}\right)}\right]\frac{TL}{\sigma}=\left(1+\frac{T\varphi_1}{\sigma}\right)\cos\beta_1-\cos\beta_0+\frac{\beta_1-\beta_0}{\ln\left(1+\frac{T\varphi_1}{\sigma}\right)}\left[\left(1+\frac{T\varphi_1}{\sigma}\right)\sin\beta_1-\sin\beta_0\right].$$

The constants are now determined by the formulas

$$c=\frac{T(\beta_1-\beta_0)}{\ln(1+T\varphi_1/\sigma)},\ c_1=L-\frac{\sigma+T\varphi}{T^2+c^2}(T\cos\beta_1+c\sin\beta_1),$$

$$c_2=\frac{\sigma}{T^2+c^2}(c\cos\beta_0-T\sin\beta_0),\ c_3=\beta_0.$$

For instance for $\beta_0 = 0$ and $\beta_1 = \pi/2$ we present the numerical values $c = -1.507T$, $c_1 = -0.306\sigma/T$, $c_2 = -0.46\sigma/T$, $c_3 = 0$.

Let us note that the boundary layer functions that appear in the neighborhood of the solid boundaries S and the contact points of S and Γ are not in (2.3). The asymptotic expansions are of more complex nature in the neighborhood of the contact points. The boundary layer functions in these domains yield a contribution to the equation of the free boundary only in higher approximations, starting with the second and, consequently, are not presented here.

Example 2. Let us consider the case when the fluid abuts on a solid vertical wall $x = 0$ only on the one side for $x > 0$. Let a constant tangential stress $T =$ const > 0 directed from the wall act on the free boundary Γ. We select the measurement of the parameter φ from the wall toward increasing x. Exactly as in the preceding example, $\varphi_0 = f_0(s) = \gamma = 0$. We introduce the Oz axis so that $\zeta(\varphi) = 0$ for $\varphi = \infty$, then $c = 0$ in (2.1). We assume that $|\zeta'(\varphi)|$ is small; then the linearized equation (2.1) has the following solution for $g \neq 0$:

$$\zeta_0 = c_0 \sqrt{\sigma + T\varphi} K_1 \left(2 \sqrt{\frac{\rho g}{T^2} (\sigma + T\varphi)} \right)$$

[$K_1(t)$ is the modified Bessel function]. The constant c_0 is determined easily by the value of the wetting angle at the point of fluid contact with the wall.

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NONLINEAR CRITICAL LAYER AND FORMATION OF LINEAR VORTICES WITH REACTION OF WAVES IN SHEAR FLOWS

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UDC 532.526

Recently in hydromechanics there has been a considerable increase in interest in the problem of transition from laminar to turbulent flow [1-3]. Of considerable importance for explaining processes occurring during transition to turbulence in shear flows is analysis of the nonlinear structures occurring as a result of the development of hydrodynamic instability. Experiments show that in boundary flows such as a boundary layer and Poiseuille flow, occurrence of turbulence is connected with formation of Λ-vortices characterized by a considerable linear (in relation to flow direction) component of vorticity [4-9]. In [10] attention was drawn to the related connection of these vortices with large-scale bounded structures observed in the region near the wall of developed turbulent flow.

The theory of Benney and Lin [11, 12] connects oscillation of linear vorticity in transitional flow with an increase in it of pairs of inclined (three-dimensional) waves having the same phase velocity and linear components of wave vectors. The instantaneous profile of the transverse velocity determined in [12] within the framework of linear approximation demonstrates two reversals of velocity for the period of the wave, whereas in experiments [5, 10] sequences of profiles are observed with one reversal which corresponds to passage through a stationary observation point for one vortex formation in the period of the wave. In this work a study is made of essentially nonlinear vortex structures occurring in a critical layer (CL) of laminar flow with resonance reaction of two-dimensional and inclined waves increasing in it. Analysis is built up within the framework of an asymptotic approach rest-

Gor'kii. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 107-115, September-October, 1987. Original article submitted June 16, 1986.

0021-8944/87/2805-0742$12.50